OCR

Monday 19 May 2014 – Morning

A2 GCE MATHEMATICS

4729/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4729/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A football is kicked from horizontal ground with speed $20\,\text{m s}^{-1}$ at an angle of $\theta°$ above the horizontal. The greatest height the football reaches above ground level is 2.44 m. By modelling the football as a particle and ignoring air resistance, find

(i) the value of θ, **[2]**

(ii) the range of the football. **[2]**

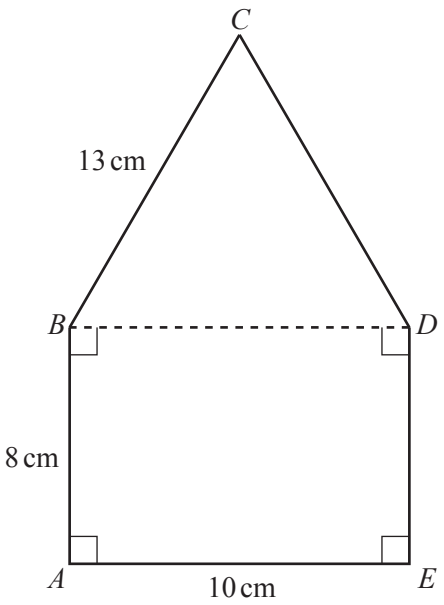
2 A uniform solid cylinder of height 12 cm and radius r cm is in equilibrium on a rough inclined plane with one of its circular faces in contact with the plane.

(i) The cylinder is on the point of toppling when the angle of inclination of the plane to the horizontal is 21°. Find r. **[3]**

The cylinder is now placed on a different inclined plane with one of its circular faces in contact with the plane. This plane is also inclined at 21° to the horizontal. The coefficient of friction between this plane and the cylinder is μ.

(ii) The cylinder slides down this plane but does not topple. Find an inequality for μ. **[2]**

3

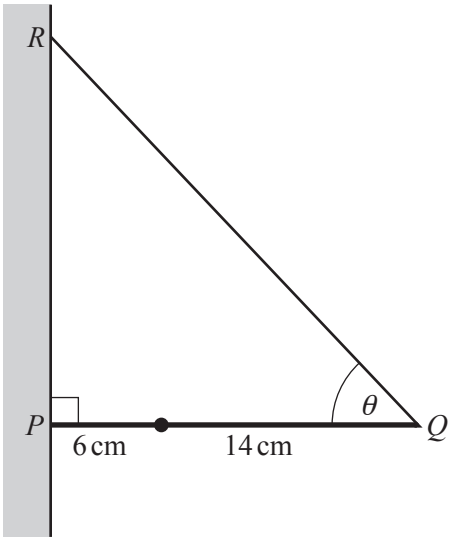


A uniform lamina $ABCDE$ consists of a rectangle $ABDE$ and an isosceles triangle BCD joined along their common edge. $AB = DE = 8\,\text{cm}$, $AE = BD = 10\,\text{cm}$ and $BC = CD = 13\,\text{cm}$ (see diagram).

(i) Find the distance of the centre of mass of the lamina from AE. **[5]**

(ii) The lamina is freely suspended from B and hangs in equilibrium. Calculate the angle that BD makes with the vertical. **[3]**

4



A uniform rod PQ has weight 18 N and length 20 cm. The end P rests against a rough vertical wall. A particle of weight 3 N is attached to the rod at a point 6 cm from P. The rod is held in a horizontal position, perpendicular to the wall, by a light inextensible string attached to the rod at Q and to a point R on the wall vertically above P, as shown in the diagram. The string is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$. The system is in limiting equilibrium.

(i) Find the tension in the string. **[3]**

(ii) Find the magnitude of the force exerted by the wall on the rod. **[4]**

(iii) Find the coefficient of friction between the wall and the rod. **[2]**

5 **(i)** A car of mass 800 kg is moving at a constant speed of $20\,\text{m s}^{-1}$ on a straight road down a hill inclined at an angle α to the horizontal. The engine of the car works at a constant rate of 10 kW and there is a resistance to motion of 1300 N. Show that $\sin\alpha = \frac{5}{49}$. **[4]**

(ii) The car now travels up the same hill and its engine now works at a constant rate of 20 kW. The resistance to motion remains 1300 N. The car starts from rest and its speed is $8\,\text{m s}^{-1}$ after it has travelled a distance of 22.1 m. Calculate the time taken by the car to travel this distance. **[5]**

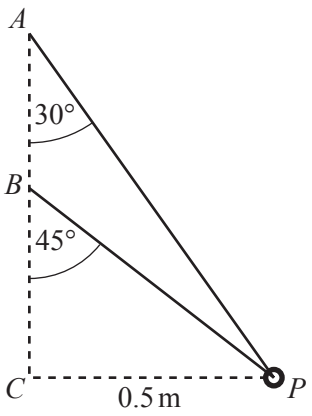
6 Two small spheres A and B, of masses $2m$ kg and $3m$ kg respectively, are moving in opposite directions along the same straight line towards each other on a smooth horizontal surface. A has speed $4\,\text{m s}^{-1}$ and B has speed $2\,\text{m s}^{-1}$ before they collide. The coefficient of restitution between A and B is 0.4.

(i) Find the speed of each sphere after the collision. **[6]**

(ii) Find, in terms of m, the loss of kinetic energy during the collision. **[4]**

(iii) Given that the magnitude of the impulse exerted on A by B during the collision is 2.52 N s, find m. **[3]**

7



A small smooth ring P of mass 0.4 kg is threaded onto a light inextensible string fixed at A and B as shown in the diagram, with A vertically above B. The string is inclined to the vertical at angles of 30° and 45° at A and B respectively. P moves in a horizontal circle of radius 0.5 m about a point C vertically below B.

(i) Calculate the tension in the string. **[3]**

(ii) Calculate the speed of P. **[3]**

The end of the string at B is moved so both ends of the string are now fixed at A.

(iii) Show that, when the string is taut, AP is now 0.854 m correct to 3 significant figures. **[2]**

P moves in a horizontal circle with angular speed 3.46 rad s^{-1}.

(iv) Find the tension in the string and the angle that the string now makes with the vertical. **[4]**

8 A child is trying to throw a small stone to hit a target painted on a vertical wall. The child and the wall are on horizontal ground. The child is standing a horizontal distance of 8 m from the base of the wall. The child throws the stone from a height of 1 m with speed 12 m s^{-1} at an angle of 20° above the horizontal.

(i) Find the direction of motion of the stone when it hits the wall. **[6]**

The child now throws the stone with a speed of V m s^{-1} from the same initial position and still at an angle of 20° above the horizontal. This time the stone hits the target which is 2.5 m above the ground.

(ii) Find V. **[6]**

Question			Answer	Marks	Guidance
1	(i)		$(20\sin\theta)^2 - 2g(2.44) = 0$ $\theta = 20.2$	M1 A1 **[2]**	Use $v^2 = u^2 + 2as$ vertically with $v = 0$ $\theta = 20.22908\ldots$
	(ii)		20 sin cv(θ)$t - 1/2gt^2 = 0$ AND range = 20 cv(t) cos cv(θ) Range = 26.5 m	M1 A1 **[2]**	Use $s = ut + \frac{1}{2}at^2$ vertically with $s = 0$ OR use $v = u + at$ and doubles t AND horizontally with time found from vertical. (t = 1.4113… s or 1.4093…s (from 20.2)) Range = 26.48541… m or 26.45387….m (from 20.2)
		OR	$\frac{20^2 \sin(2 \times \text{cv}(\theta))}{g}$ Range = 26.5 m	M1 A1 **[2]**	Use of range formula Range = 26.48541… m or 26.45387….m (from 20.2)
2	(i)		 $r/6 = \tan 21$ $r = 2.3(0)$	M1 A1 A1 **[3]**	Attempt to use trigonometry to form equation for r $r = 2.30318\ldots$
	(ii)		$\mu <$ cv(r)/6 or $\mu mg\cos 21 < mg\sin 21$ $\mu < 0.384$ or tan 21	M1 A1 **[2]**	Attempt comparison between weight comp and max friction. $\mu < 0.38386\ldots$ or 0.38333…. (from 2.3); allow ≤
3	(i)		CoM of triangle = $^1/_3$ x cv(12) from BD $(80 + 60)x_G$ $= 4(80) + 12(60)$ $x_G = 7.43$ cm	B1 M1 A1 A1 A1 **[5]**	OR $^2/_3$ x cv(12) from C. CoM of triangle Table of values idea 7.42857… or $^{52}/_7$ cm
	(ii)		$\tan\theta = (8 - x_G)/5$ $\tan\theta = 0.5714\ldots/5$ $\theta = 6.52°$	M1 A1ft A1 **[3]**	Using tan to find a relevant angle ft their x_G to target angle with the vertical 6.5198… Allow 6.5(0) from $x_G = 7.43$

Question			Answer	Marks	Guidance
4	(i)		$18(10) - T(20\sin\theta) + 3(6) = 0$ $T = 16.5$ N	M1 A1 A1 **[3]**	Moments about P Need a value for $\sin\theta$ or θ Exact
	(ii)		$X = T\cos\theta$ $Y + T\sin\theta - 18 - 3 = 0$ $R = \sqrt{(13.2^2 + 11.1^2)} = 17.2$ N	B1ft M1 A1ft A1 **[4]**	ft candidates value of T. Resolve horizontally ($X = 13.2$ N) or moments; Need a value for $\cos\theta$ or θ Resolve vertically or moments ft candidates value of T. $Y = 11.1$ N; Need a value for $\sin\theta$ or θ $R = 17.2467\ldots$
	(iii)		$\mu = \text{cv}(\lvert Y\rvert)/\text{cv}(\lvert X\rvert) = 11.1/13.2$ $\mu = 0.841$	M1 A1 **[2]**	Use of $Fr = \mu R$ $\mu = 0.8409\ldots$; allow $^{37}/_{44}$
5	(i)		Driving Force = 10000/20 (= 500) $\text{cv}(10000/20) - 1300 + 800g\sin\alpha = 0$ $\sin\alpha = 5/49$	B1 M1 A1 A1 **[4]**	 Attempt at N2L with 3 terms **AG** at least one more line of correct working (at least e.g. $-800+800g\sin\alpha=0$); allow verification (e.g. 500 – 1300 + 800 = 0)
	(ii)		$800(22.1)g\sin\alpha$ $800(22.1)g\sin\alpha + 1300(22.1) + \frac{1}{2}(800)(8^2)$ $t = 3.6(0)$ s	B1 M1 A1 M1 A1 **[5]**	Work done against weight; Need a value for $\sin\alpha$ or α Total work done, 3 terms needed Need a value for $\sin\alpha$ or α; (72010 J) Time = work done(from at least one correct energy term)/power 'Exact' is 3.6005
6	(i)		$(2m)(4) - (3m)(2) = 2mv_A + 3mv_B$ $(v_B - v_A)/(4 - -2) = 0.4$ Speed $A = 1.04$ m s^{-1}, Speed $B = 1.36$ m s^{-1}	*M1 A1 *M1 A1 Dep**M1 A1 **[6]**	Attempt at use of conservation of momentum Attempt at use of coefficient of restitution Solving for v_A and v_B Final answers must be positive

Question			Answer	Marks	Guidance
	(ii)		Energy before = ½(2*m*)(4^2) + ½(3*m*)(2^2) Energy after = ½(2*m*)(1.04^2) + ½(3*m*)(1.36^2) 22*m* – 3.856*m* 18.1*m*	B1ft B1ft M1 A1 **[4]**	Energy before or Loss in *A*'s KE Energy after or Loss in *B*'s KE Difference of total OR sum of differences (total kinetic energy must decrease) 18.144*m* (Exact)
		OR	$\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(1-e^2)A^2$ $\frac{1}{2}\frac{(2m)(3m)}{2m+3m}(1-0.4^2)(4+2)^2$ 18.1*m*	*B1 Dep*M1 A1 A1 **[4]**	Loss of kinetic energy formula, where *A* = approach speed Substitution of values into quoted formula 18.144*m* (Exact)
	(iii)		2*m*(4) – 2*m*(−1.04) = 2.52 *m* = 0.25	M1 A1ft A1 **[3]**	Attempt at change in momentum and equate to impulse. Must use 2m or 3m Or 3*m*(2) – 3*m*(−1.36) = 2.52 Exact
7	**(i)**		*T*cos30 + *T*cos45 = 0.4*g* *T* = 2.49 N	M1 A1 A1 **[3]**	Resolve vertically (3 terms); may be different *T*'s at this stage *T* = 2.4918.…
	(ii)		cv(*T*)sin30 + cv(*T*)sin45 = 0.4*v*^{2}/0.5 *v* = 1.94 m s^{-1}	M1 A1 A1 **[3]**	Resolve horizontally (3 terms); may be different *T*'s at this stage Or use acceleration = 0.5ω^2 *v* = 1.93904…
	(iii)		$(2AP =)\frac{0.5}{\sin 45}+\frac{0.5}{\sin 30}$ *AP* = 0.854 m	M1 A1 **[2]**	Reasonable attempt to use trigonometry to find total length of string **AG** (*AP* = 0.85355…m)

Question			Answer	Marks	Guidance
	(iv)		$2T\sin\theta = 0.4(0.854\sin\theta)(3.46^2)$	M1	θ angle with vertical. Resolve horizontally. Allow with T only. r = component of 0.854
			$T = 2.04$ N	A1	$T = 2.04474\ldots$ N using $AP = 0.854$ m, $T = 2.04367\ldots$ N using exact AP
			$2T\cos\theta = 0.4g$	M1	θ angle with vertical. Resolve vertically. Allow with T only
			$\theta = 16.5°$ or $16.6°$	A1	$\theta = 16.55377\ldots°$ using $AP = 0.854$ m, $\theta = 16.4526\ldots°$ using exact AP
				[4]	**SC** M1A0M1A1 for use of T instead of 2T throughout
8	**(i)**		$v_x = 12\cos20$	*B1	11.27631…..
			$8 = 12t\cos20$	B1	Using suvat to find expression in t only. ($t = 0.70945\ldots$)
				*M1	Attempt at use of $v = u + at$
			$v_y = 12\sin20 - g$cv(t)	A1	-2.84838……
			$\tan\theta = v_y / v_x$	Dep**M1	Use trig to find a relevant angle
			14.2° below horizontal	A1	14.1763… (75.8° downward vertical)
				[6]	
	(ii)		$8 = Vt\cos20$	B1	
				*M1	Attempt at use of $s = ut + ½\ at^2$
			$1.5 = Vt\sin20 - gt^2/2$	A1	
			Eliminate t	dep*M1	OR Eliminate V and solve for t
			Attempt to solve a quadratic for V	dep*M1	AND Sub value for t and solve for V
			$V = 15.9$	A1	$V = 15.8606\ldots$
				[6]	
		OR	$y = x\tan\theta - gx^2\sec^2\theta/2u^2$	*B1	Use equation of trajectory
			Substitute values for y, x, θ	dep*M1	
			$1.5 = 8\tan20 - g8^2\sec^2 20/2V^2$	A1	
			Attempt to solve a quadratic for V	dep*M2	SC M1 for solving for V^2
			$V = 15.9$	A1	$V = 15.8606\ldots$
				[6]	